

June 2004

Energy from the Bulk through Parametric Resonance

F. K. Diakonos, E. N. Saridakis and N. Tetradis

Department of Physics, University of Athens, Zographou 157 71, Greece

Abstract

We demonstrate that energy can be transferred very rapidly from the bulk to the brane through parametric resonance. In a simple realization, we consider a massless bulk field that interacts only with fields localized on the brane. We assume an initial field configuration that has the form of a wave-packet moving towards the brane. During the reflection of the wave-packet by the brane the localized fields can be excited through parametric resonance. The mechanism is also applicable to bulk fields with a potential. The rapid energy transfer can have important cosmological and astrophysical implications.

PACS numbers: 11.10.Kk, 04.50, 98.80.Cq

Introduction: The idea that our Universe is a defect embedded in a higher-dimensional bulk space is old [1]. The Randall-Sundrum model [2] provides a realization of this idea. The defect is a four-dimensional hypersurface (a 3-brane) in a five-dimensional bulk with negative cosmological constant (AdS space). The geometry is non-trivial (warped) along the fourth spatial dimension, so that an effective localization of low-energy gravity takes place near the brane. The matter is assumed to be concentrated only on the brane. For low matter densities the cosmological evolution on the brane becomes typical of a Friedmann-Robertson-Walker Universe [3, 4].

Of particular interest in the scenario of a brane Universe is the possibility of exchange of energy between the defect and the bulk space. For this to occur the bulk must contain some matter component in addition to the negative cosmological constant. The presence of matter in the bulk affects the expansion on the brane. For an observer on the brane the modification can be attributed to “mirage” matter components [5]. The energy exchange can modify the cosmological evolution as well [6]. In particular, the energy transfer from the bulk to the brane can lead to periods of accelerated expansion on the brane [7].

In this letter we concentrate on the mechanism that can lead to a rapid energy transfer from the bulk to the brane. We omit the gravitational effects and study a simplified system that consists of a flat five-dimensional bulk space with a four-dimensional brane as its boundary. In the bulk we assume the presence of a field that interacts only with fields localized on the brane. Our aim is to demonstrate that energy can be transferred very efficiently from the bulk to the brane fields through a process that resembles very strongly the parametric resonance [8, 9]. The incorporation of gravitational effects is technically complicated and will be left for a future publication.

The model: We consider a massless bulk field ϕ that interacts with a field χ localized on the brane. We neglect gravitational effects and assume that the system is described by the action

$$S = \int d^4x dy \left[\partial^M \phi \partial_M \phi + \delta(y) (\partial^\mu \chi \partial_\mu \chi - V(\chi, \phi)) \right], \quad (1)$$

where $M = 0, 1, 2, 3, 4$ and $\mu = 0, 1, 2, 3$. In order to keep our discussion simple we identify the half-space $y < 0$ with the half-space $y > 0$, in complete analogy to ref. [2]. In this way the brane forms the boundary of the bulk space. We assume that the potential $V(\chi, \phi)$ has the form

$$V(\chi, \phi) = \frac{1}{2} m^2 \chi^2 + \frac{1}{2} g \chi^2 \phi^2. \quad (2)$$

We expand the quantum field χ as

$$\chi(t, \vec{x}) = \int \frac{d^3k}{(2\pi)^3} \left(a_k f_k(t) e^{-i\vec{k}\vec{x}} + a_k^\dagger f_k^*(t) e^{i\vec{k}\vec{x}} \right), \quad (3)$$

where the mode functions f_k satisfy $f_k \dot{f}_k^* - f_k^* \dot{f}_k = i$, and the number distribution function is $n_k \equiv \langle a_k^\dagger a_k \rangle$. The equation of motion of χ gives

$$\ddot{f}_k(t) + \left(k^2 + m^2 + g\phi^2(t, 0) \right) f_k(t) = 0, \quad (4)$$

where a dot denotes a time derivative. We also have

$$\langle \chi^2 \rangle(t) = \int \frac{d^3 k}{(2\pi)^3} f_k^*(t) f_k(t) (1 + 2n_k). \quad (5)$$

We treat the field ϕ classically. We also make the simplifying assumption that the configuration of ϕ depends only on the time t and the fourth spatial coordinate y . For this assumption to be consistent with the ϕ -field evolution we replace $\chi^2(t, \vec{x})$ with $\langle \chi^2 \rangle(t)$ in the equation of motion of ϕ . This is the Hartree approximation [10] that is usually employed in the study of parametric resonance [9]. The ϕ -field equation of motion becomes

$$\ddot{\phi}(t, y) - \phi''(t, y) + g \delta(y) \phi(y, t) \langle \chi^2 \rangle(t) = 0, \quad (6)$$

where a prime denotes a y -derivative. Integrating this equation with respect to y in the interval $[-\epsilon, \epsilon]$ and taking $\epsilon \rightarrow 0$, we find

$$\phi'(t, 0^+) = -\phi'(t, 0^-) = \frac{1}{2} g \phi(t, 0) \langle \chi^2 \rangle(t). \quad (7)$$

The solution of eq. (6) is equivalent to the solution of the bulk equation

$$\ddot{\phi}(t, y) - \phi''(t, y) = 0 \quad (8)$$

with the boundary condition (7).

In the following we solve numerically the system of equations (4), (5), (7), (8) in the region $y \geq 0$. The solution for negative y is $\phi(t, -y) = \phi(t, y)$. The initial configuration $\phi(0, y)$ is arbitrary. We choose it to have the form of a wave-packet that moves towards the brane. For the mode functions of the χ -field for $t \rightarrow 0$, when $\phi(t, 0) \simeq 0$, we assume the vacuum expressions $f_k(t) = e^{-i\omega_k t} / \sqrt{2\omega_k}$, with $\omega_k = \sqrt{k^2 + m^2}$. This gives $f_k(0) = 1/\sqrt{2\omega_k}$, $\dot{f}_k(0) = -i\omega_k/\sqrt{2\omega_k}$. The evolution of $f_k(t)$ can be attributed to the creation of χ -particles on the brane through the time-varying ϕ -field. The time-dependent number distribution function can be calculated through a Bogoliubov transformation [10]. It is approximately

$$n_k(t) = \frac{1}{2} \omega_k(t) \left(|f_k(t)|^2 + \frac{|\dot{f}_k(t)|^2}{\omega_k^2(t)} \right) (2n_k + 1) - \frac{1}{2}, \quad (9)$$

with $\omega_k(t) = \sqrt{k^2 + m^2 + g\phi^2(t, 0)}$. The above expression has corrections $\sim \dot{\omega}_k$ that we have neglected by assuming that $\dot{\omega}_k(t) \ll \omega_k^2(t)$. In the limit $t \rightarrow \infty$, after the ϕ -wave-packet has been reflected by the brane and $\phi(t, 0) \simeq 0$, our approximation becomes exact. This means that the final number of χ -particles is computed exactly, even though the expression (9) is approximate when $\phi(t, 0) \neq 0$. The initial number distribution function n_k can be chosen according to our assumptions for the initial state of the physical system. For example, it can have the form of a thermal distribution [10]. We make the simpler assumption that initially there is no significant number density of χ -particles, so that $n_k = 0$.

Several approximations that we have made in deriving the evolution equations for the fields are not expected to affect the qualitative physical behaviour. For example, a term $\sim \chi^4$ in the potential of eq. (2) has been taken into account in previous studies of parametric resonance, without altering significantly the essential behaviour. The Hartree approximation becomes exact in the large- N limit, if the field χ is assumed to have N components and the Lagrangian to be invariant under an $O(N)$ symmetry. Finally, the possible presence of a potential for the bulk field is not essential either. As we shall see in the following, the necessary requirement is that the bulk field must be oscillatory at the location of the brane. Any wave solution that is reflected by the brane is capable of transferring energy onto it through parametric resonance.

An analytical solution: In order to have some intuitive understanding of the solutions of interest we derive in this section an analytical solution of eqs. (4), (5), (7), (8). We make the assumption $\phi(t, 0) = \phi_0 \cos \alpha t$. Then, eq. (4) can be put in the form of the Mathieu equation

$$\frac{d^2 f_k}{dz^2} + (A_k - 2q \cos 2z) f_k = 0, \quad (10)$$

with $z = \alpha t - \pi/2$, $A_k = (k^2 + m^2 + g\phi_0^2/2)/\alpha^2$, $q = g\phi_0^2/2\alpha^2$. An infinite number of resonance bands exist in k , within which the amplitude of f_k grows exponentially fast with time. As a result, $\langle \chi^2 \rangle(t)$ grows fast.

The form of $\phi(t, y)$ that is consistent with the above solution for χ can be determined analytically. It is given by the solution of the wave equation (8) for $y \geq 0$, with the “initial” conditions given for $y = 0$. Essentially the role of t and y is reversed compared to the standard case. The most general solution of eq. (8) is

$$\phi(t, y) = \frac{1}{2} \left[a(t+y) + a(t-y) + \int_{t-y}^{t+y} b(t') dt' \right], \quad (11)$$

where the functions $a(t)$, $b(t)$ are determined through the boundary conditions at $y = 0$. In particular, $a(t) = \phi(t, 0) = \phi_0 \cos \alpha t$ according to our assumption. The function $b(t)$ is defined through eq. (7), that gives $b(t) = \phi'(t, 0) = g\phi(t, 0) \langle \chi^2 \rangle(t)/2$.

For a given time t the field $\phi(t, y)$ has an oscillatory form with an envelope that grows as a function of y . As time increases the slope of the envelope grows. This behaviour is a consequence of our assumption that ϕ has a constant amplitude ϕ_0 at $y = 0$. The growth of $\langle \chi^2 \rangle(t)$ induces a backreaction on the evolution of ϕ that tends to reduce its amplitude. This effect can be compensated only if the amplitude of the bulk oscillations of ϕ increases with time.

The numerical solution: The analytical solution indicates that the rapid transfer of energy from the bulk to the brane through parametric resonance is possible. However, the solution was constructed by assuming the form of the ϕ -field at the location of the brane and deducing its form in the bulk. Here we would like to start by defining initial conditions for the system, such that the deviation of ϕ from zero is significant only away

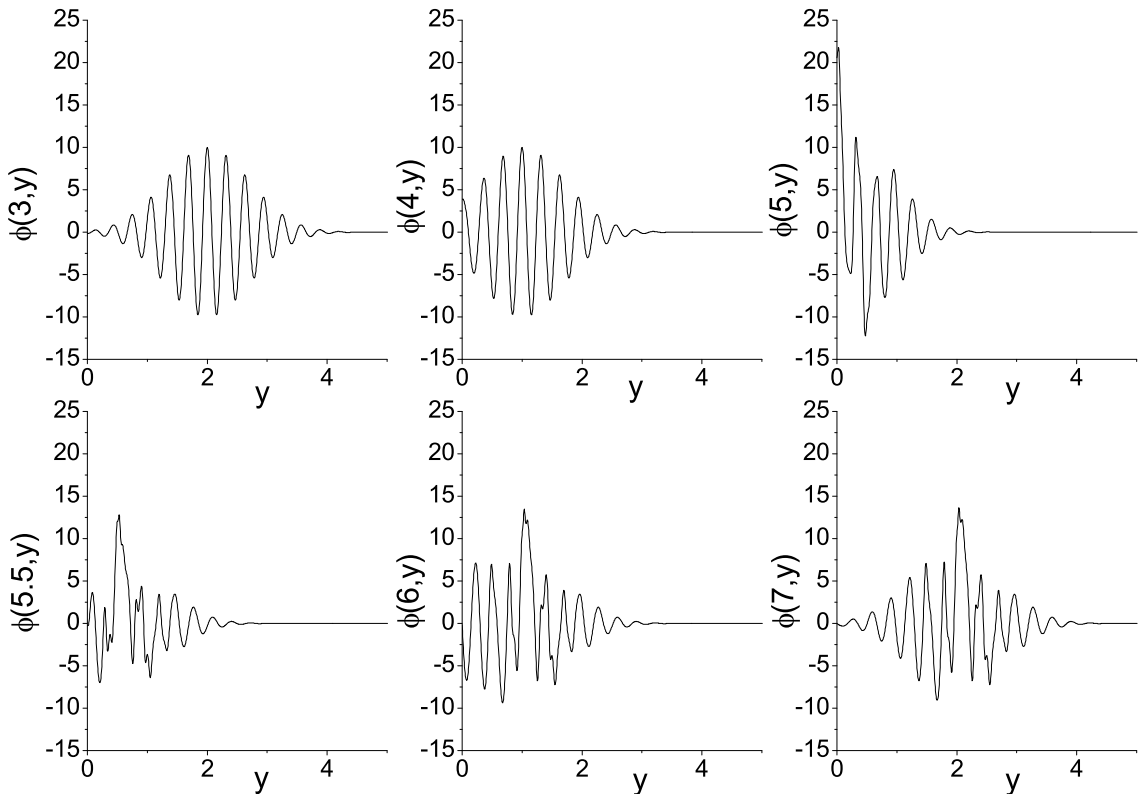


Figure 1: *The form of the bulk field at various times.*

from the brane. Subsequently, the ϕ -field perturbation moves close to the brane and leads to the excitation of the χ field. This calculation cannot be performed analytically because of the difficulty in implementing the boundary condition (7). For this reason we solve the system of equations (4), (5), (7), (8) numerically.

For early times we consider a configuration for the ϕ -field given by

$$\phi(t, y) = \phi_0 \exp\left(-\frac{(y - y_0 + t)^2}{\sigma^2}\right) \cos[\alpha(y - y_0 + t)]. \quad (12)$$

It corresponds to a wave-packet of fundamental frequency α and width σ , centered at $y = y_0 - t$, that moves towards the brane located at $y = 0$ with the speed of light. In the following we study the evolution of a wave-packet with $\phi_0 = 10$, $y_0 = 5$, $\sigma = 1$, $\alpha = 20$. We use the values $m^2 = 1$, $g = 10$ for the parameters in the potential of eq.(2). Essentially, we express all dimensionful quantities in units of m . We also assume a negligible initial

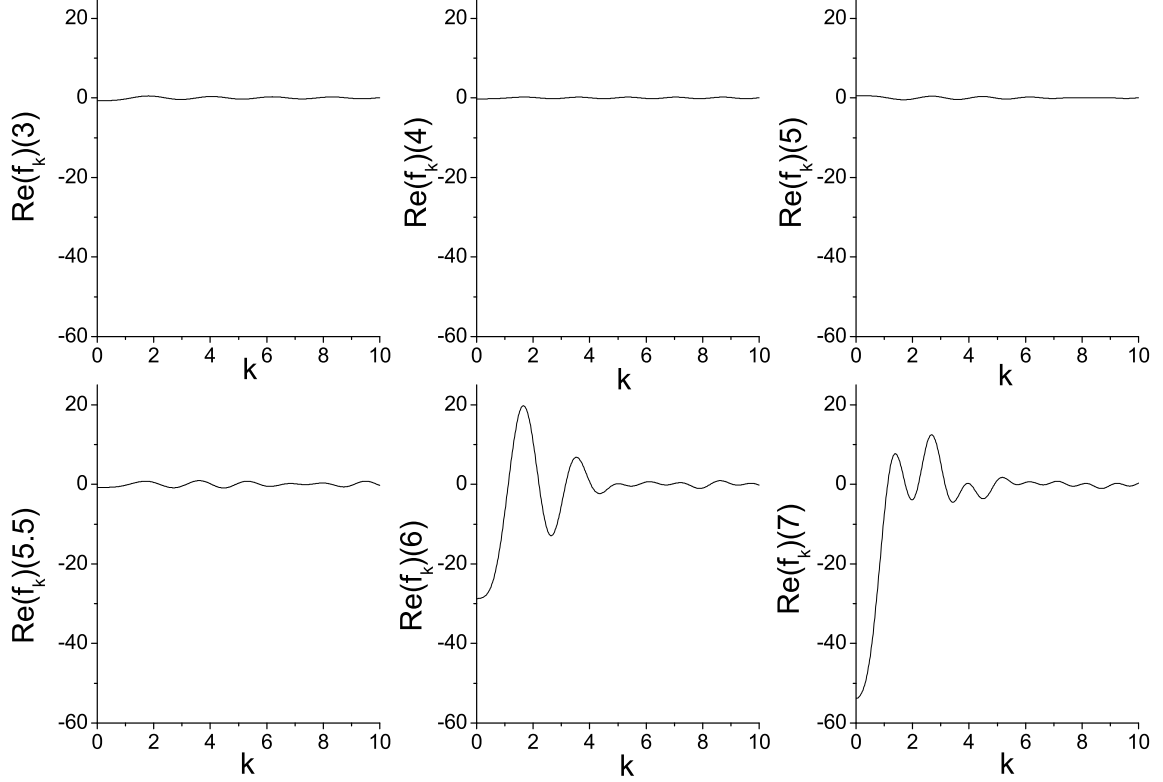


Figure 2: *The real part of the mode function f_k of the brane field at various times.*

particle number density: $n_k = 0$. The total number density of χ -particles and their energy density are given by

$$\frac{N(t)}{V} = \int \frac{d^3k}{(2\pi)^3} n_k(k), \quad \frac{E(t)}{V} = \int \frac{d^3k}{(2\pi)^3} \omega_k(t) n_k(k), \quad (13)$$

with $n_k(t)$ given by eq. (9). The momentum integrations in these equations, as well as in eq. (5), have ultraviolet divergences. We regulate them through an explicit cutoff $\Lambda = 10$. Moreover, the time-evolution of the χ -field is expected to display the well-known ultraviolet problem of classical systems. (The Hartree approximation in the leading order leads to essentially classical evolution.) The high-frequency modes become excited at sufficiently large times and the ultraviolet contributions dominate in the expressions for the various observables. In our simplified study we omit all χ -field self-interaction terms. As a result, the ultraviolet modes can be excited only during the finite interval that the wave-packet interacts with the brane. For our choice of parameters, no significant

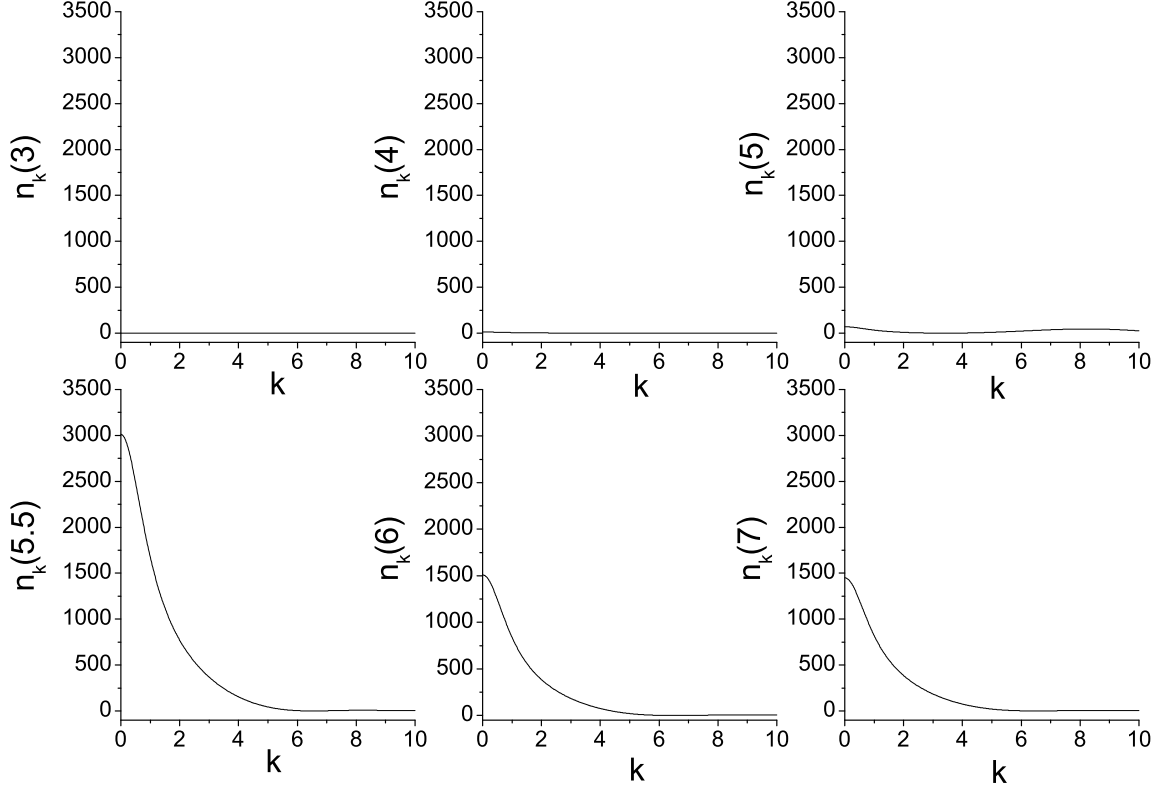


Figure 3: *The number distribution function n_k on the brane at various times.*

excitation of these modes takes place.

In fig. 1 we display the evolution of the wave-packet. At $t = 0$ it is located away from the brane. Subsequently it moves towards the brane and interacts with it during the time interval $3 < t < 7$. Eventually it gets reflected with a distorted profile. At times $t \simeq 5$ the maximum of $\phi(t, 0)$ is $\simeq 2\phi_0$. The reason is that, because of our assumption of reflection symmetry around $y = 0$, there is another wave-packet interacting with the brane from the left.

In fig. 2 we depict the evolution of the mode functions $f_k(t)$ of the χ -field. We observe that they increase strongly in the region of small k through parametric resonance. The mode functions with k near the cutoff $\Lambda = 10$ are not excited significantly during the time interval that we consider. This implies that we can trust the predicted physical behaviour.

In fig. 3 we display the number distribution function $n_k(t)$ as given by eq. (9). We observe that during the time interval $3 < t < 7$ the low-frequency modes are excited. The

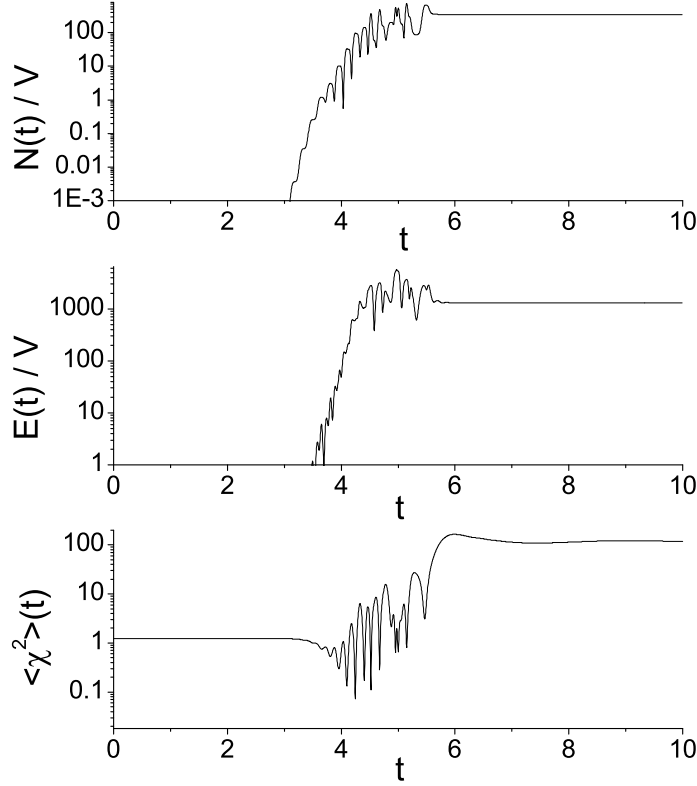


Figure 4: *The total number density of χ -particles, the energy density carried by them and the dispersion of the χ -field, as a function of time.*

quantity $n_k(t)$ displays some oscillatory behaviour with time, but on the average it grows fast. For $t > 7$ the wave-packet moves away from the brane and the distribution function becomes constant. This is a consequence of the omission of any χ -field self-interactions in our simplified treatment.

In fig. 4 we plot the total number density of χ -particles, the energy density carried by them and the dispersion of the χ -field. All these quantities display a rapid increase in the time interval $3 < t < 7$, while they remain constant at later times when the wave-packet has moved away from the brane.

The intensity of the parametric resonance depends on the parameters of the model. We can use the Mathieu equation (10) in order to obtain some intuition. The amplification is more pronounced if the system is in the region of broad parametric resonance. This

requires $g\phi_0^2/m^2 \gtrsim 1$, so that $A_k \gtrsim q$. The parameters we chose satisfy this constraint. The time dependence of $n_k(t)$ in fig. 3 is typical of a system in the region of broad parametric resonance [9].

The resonance effects could be further enhanced if the effective mass became zero for a certain value of χ . This would require $m^2 < 0$ and would imply that the ϕ -field determines the χ expectation value. We have used a simpler model, in which the bulk plays only a minor role in the determination of the vacuum state of the brane fields, while it can affect their dynamical evolution.

Despite the similarity we described above, a careful analysis indicates that the Mathieu equation is not strictly relevant if the bulk field has the form of a wave-packet. The reason is that the function $\phi(t, 0)$ that appears in eq. (4) describes an oscillation with an amplitude that is non-zero only within a finite time interval. The Fourier transform of this function is non-zero over a large range of frequencies, and not for a unique frequency as in the Mathieu equation. We expect a qualitative behaviour similar to the one predicted by the Lamé equation. In agreement with this conclusion, we do not find any parameter range that resembles the region of narrow parametric resonance. The numerical analysis of the range $g\phi_0^2/m^2 \lesssim 1$ leads to an evolution very similar to the one depicted in figs. 1–4. The only difference is that the number density of the produced χ -particles and the total energy transferred to the brane fall very fast with decreasing $g\phi_0^2/m^2$. When $g\phi_0^2/m^2 \ll 1$ the resonance effects become negligible. Similarly, significant excitation of the χ -modes is possible only if the fundamental frequency α of the ϕ wave-packet is not much smaller than the mass m .

Cosmological and astrophysical implications: The rapid transfer of energy from the bulk to the brane can affect the cosmological evolution. If the χ -field is coupled to lighter brane particles the energy can be transferred to them through the decay of the χ -modes. This would result in a period like preheating [8, 9]. The two mechanisms are so similar that the process we have described could be characterized as preheating by a bulk source. There are several advantageous aspects for this type of preheating:

- a) The form of the source is less constrained than in the conventional scenario. In particular, the field ϕ need not be homogeneous. In the example we discussed it has the form of a wave-packet that depends only on the extra spatial coordinate. There is no need for a specially chosen potential for ϕ . In our example the potential was assumed to be zero.
- b) If ϕ is not the field responsible for the brane inflation, the time of preheating and the amount of the released energy are not constrained. In our example, for $g\phi_0^2/m^2 \gtrsim 1$ the energy density in the χ -modes exceeds m^4 . For large m significant amounts of energy can be deposited on the brane.

On the other hand, this scenario requires an additional non-compact spatial dimension. A picture consistent with observations can be obtained only by generalizing the Randall-Sundrum model [2] in a cosmological context. The graviton localization near the brane leads to a low-energy cosmological evolution typical of a four-dimensional Universe [3, 4]. The effect of the energy influx on the cosmological evolution can be more complicated than the mere increase of the brane energy density. It is possible that an era of cosmological

acceleration can take place [7]. A detailed investigation of this issue must include the solution of the Einstein equations for the generalized warped geometry of the Randall-Sundrum model [4]. A numerical study of such a system is technically difficult. The only existing study has considered a two-brane scenario without brane fields [11]. The numerical investigation of the scenario with brane fields is in progress.

Another manifestation of the mechanism we described could be related to the energy spectrum of high energy cosmic rays. It has been argued that these could be produced in the decays of long-lived heavy particles of cosmological origin [12, 13]. Our mechanism provides an alternative possibility: The decaying heavy particles could correspond to the χ -modes that are produced during the energy transfer from the bulk to the brane. In order to provide an explanation for the highest energy cosmic rays, the mass of the χ -particles must be $m \gtrsim 10^{13}$ GeV [12, 13]. The production of so heavy particles is possible only if the fundamental frequency α of the wave-packet is of the same order of magnitude.

The total amount of transferred energy is controlled by the combination $g\phi_0$, where g is the strength of the coupling between the bulk and brane fields and ϕ_0 the maximum amplitude of the wave-packet. The total amount of energy stored in χ -particles drops very fast for $g\phi_0 \ll m$, with m the mass of the χ -field. In this range the resonance effects become negligible. Moreover, the dilution of the energy density through the possible accelerated expansion on the brane because of the energy influx has not been taken into account in our discussion. The combination of the two factors indicates that the total amount of transferred energy can be much smaller than m^4 , so as not to disturb the conventional cosmological picture, even though the energy will be distributed in heavy particles of mass m . The quantitative analysis of these issues will be possible only through the numerical solution of the Einstein equations.

As a final comment we mention that the role of the bulk field can be played by the gravitational sector of the theory. It is possible that a perturbation of the bulk metric that has the form of a gravitational wave interacting with the brane can result in the excitation of heavy brane particles. This is an effect opposite to the production of bulk gravitons by the brane matter [14].

Acknowledgements: The work of E. N. Saridakis was supported by the Greek State Scholarship's Foundation (IKY). The work of N. Tetradis was partially supported through the RTN contract HPRN-CT-2000-00148 of the European Union. The authors acknowledge financial support through the research programs "Pythagoras" and "Kapodistrias".

References

- [1] K. Akama, Lect. Notes Phys. **176** (1982) 267 [arXiv:hep-th/0001113];
V. A. Rubakov and M. E. Shaposhnikov, Phys. Lett. B **125** (1983) 136.
- [2] L. Randall and R. Sundrum, Phys. Rev. Lett. **83** (1999) 3370 [arXiv:hep-th/9905221]; Phys. Rev. Lett. **83** (1999) 4690 [arXiv:hep-th/9906064].

- [3] P. Binetruy, C. Deffayet and D. Langlois, Nucl. Phys. B **565** (2000) 269 [arXiv:hep-th/9905012];
P. Binetruy, C. Deffayet, U. Ellwanger and D. Langlois, Phys. Lett. B **477** (2000) 285 [arXiv:hep-th/9910219];
C. Csaki, M. Graesser, C. F. Kolda and J. Terning, Phys. Lett. B **462** (1999) 34 [arXiv:hep-ph/9906513];
J. M. Cline, C. Grojean and G. Servant, Phys. Rev. Lett. **83** (1999) 4245 [arXiv:hep-ph/9906523];
P. Kraus, JHEP **9912** (1999) 011 [arXiv:hep-th/9910149].
- [4] P. Brax and C. van de Bruck, Class. Quant. Grav. **20** (2003) R201 [arXiv:hep-th/0303095];
R. Maartens, arXiv:gr-qc/0312059.
- [5] A. Kehagias and E. Kiritsis, JHEP **9911** (1999) 022 [arXiv:hep-th/9910174];
A. Hebecker and J. March-Russell, Nucl. Phys. B **608** (2001) 375 [arXiv:hep-ph/0103214].
- [6] C. van de Bruck, M. Dorca, C. J. A. Martins and M. Parry, Phys. Lett. B **495** (2000) 183 [arXiv:hep-th/0009056];
H. A. Chamblin and H. S. Reall, Nucl. Phys. B **562** (1999) 133 [arXiv:hep-th/9903225];
C. Barcelo and M. Visser, Phys. Rev. D **63** (2001) 024004 [arXiv:gr-qc/0008008];
K. i. Maeda and D. Wands, Phys. Rev. D **62** (2000) 124009 [arXiv:hep-th/0008188];
A. Mennim and R. A. Battye, Class. Quant. Grav. **18** (2001) 2171 [arXiv:hep-th/0008192];
D. Langlois, L. Sorbo and M. Rodriguez-Martinez, Phys. Rev. Lett. **89** (2002) 171301 [arXiv:hep-th/0206146];
E. Leeper, R. Maartens and C. F. Sopuerta, Class. Quant. Grav. **21** (2004) 1125 [arXiv:gr-qc/0309080].
- [7] E. Kiritsis, G. Kofinas, N. Tetradis, T. N. Tomaras and V. Zarikas, JHEP **0302** (2003) 035 [arXiv:hep-th/0207060];
N. Tetradis, Phys. Lett. B **569** (2003) 1 [arXiv:hep-th/0211200];
Y. S. Myung and J. Y. Kim, Class. Quant. Grav. **20** (2003) L169 [arXiv:hep-th/0304033];
T. N. Tomaras, arXiv:hep-th/0404142.
- [8] L. Kofman, A. D. Linde and A. A. Starobinsky, Phys. Rev. Lett. **73** (1994) 3195 [arXiv:hep-th/9405187];
Y. Shtanov, J. Traschen and R. H. Brandenberger, Phys. Rev. D **51** (1995) 5438 [arXiv:hep-ph/9407247];
D. Boyanovsky, H. J. de Vega, R. Holman and J. F. Salgado, Phys. Rev. D **54** (1996) 7570 [arXiv:hep-ph/9608205];
D. Boyanovsky, H. J. de Vega and R. Holman, arXiv:hep-ph/9701304.
- [9] L. Kofman, A. D. Linde and A. A. Starobinsky, Phys. Rev. D **56** (1997) 3258 [arXiv:hep-ph/9704452].

- [10] F. Cooper, Y. Kluger, E. Mottola and J. P. Paz, Phys. Rev. D **51** (1995) 2377 [arXiv:hep-ph/9404357].
- [11] J. Martin, G. N. Felder, A. V. Frolov, M. Peloso and L. Kofman, Phys. Rev. D **69** (2004) 084017 [arXiv:hep-th/0309001].
- [12] V. Berezhinsky, M. Kachelriess and A. Vilenkin, Phys. Rev. Lett. **79** (1997) 4302 [arXiv:astro-ph/9708217];
V. A. Kuzmin and V. A. Rubakov, Phys. Atom. Nucl. **61**, 1028 (1998) [Yad. Fiz. **61**, 1122 (1998)] [arXiv:astro-ph/9709187].
- [13] V. Kuzmin and I. Tkachev, JETP Lett. **68** (1998) 271 [arXiv:hep-ph/9802304];
Phys. Rev. D **59** (1999) 123006 [arXiv:hep-ph/9809547];
D. J. H. Chung, E. W. Kolb and A. Riotto, Phys. Rev. D **60** (1999) 063504 [arXiv:hep-ph/9809453]; Phys. Rev. Lett. **81** (1998) 4048 [arXiv:hep-ph/9805473];
D. J. H. Chung, P. Crotty, E. W. Kolb and A. Riotto, Phys. Rev. D **64** (2001) 043503 [arXiv:hep-ph/0104100].
- [14] G. F. Giudice, R. Rattazzi and J. D. Wells, Nucl. Phys. B **544** (1999) 3 [arXiv:hep-ph/9811291].